

LARGE DEFORMATION ANALYSIS IN GEOMECHANICS USING ADAPTIVE FINITE ELEMENT METHODS

By

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DECLARATION

“This Thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying subject to the provisions of the Copyright Act 1968.”

(Signed)

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CONTENTS

DECLARATION	i
ACKNOWLEDGEMENTS	ii
Abstract	vii
Preface	x
CHAPTER 1- INTRODUCTION	1
1.1. Motivation	2
1.2. Objectives	4
1.3. Outline	5
CHAPTER 2- FINITE ELEMENTS AND ADAPTIVITY	8
2.1. Introduction	9
2.2. Literature review	11
2.3. Finite element method	18
2.4. Adaptive finite element methods.....	20
2.4.1. p -adaptive FE method	21
2.4.2. r -adaptive FE method.....	22
2.4.3. h -adaptive FE method	23
2.4.4. hp -adaptive FE method	24
2.5. Governing equations.....	25
2.5.1. Equilibrium equation.....	26

2.5.2. Constitutive equations.....	29
CHAPTER 3- H-ADAPTIVITY AND ITS ASPECTS	34
3.1. Introduction	35
3.2. H-adaptive finite element method	35
3.3. Error estimation.....	37
3.3.1. Recovery based error estimation.....	39
3.4. Mesh generation	43
3.4.1. Structured mesh generation.....	44
3.4.2. Unstructured mesh generation.....	44
3.5. Remapping.....	49
CHAPTER 4- IMPLEMENTATION OF H-ADAPTIVITY	51
4.1. Introduction	52
4.2. Error estimator.....	55
4.3. The mesh generator	58
4.3.1. Pre-processing and analysis	60
4.4. Remapping.....	61
4.5. Other aspects of implementation	64
4.5.1. Memory management.....	64
4.5.2. Ordering optimisation	65
4.5.3. Input file.....	66
CHAPTER 5- H-ADAPTIVITY FOR CONTACT AND DYNAMIC ANALYSIS	69
5.1. Introduction	70

5.2. Literature review	71
5.3. finite element Formulation	73
5.3.1. Momentum equation and its discretisation	64
5.3.2. Time Integration.....	75
5.3.3. Energy absorbing boundaries.....	79
5.3.4. Definition of contact	81
5.3.5. Retrieving dynamic equilibrium	94
5.4. H-adaptivity and dynamic/contact analysis.....	95
CHAPTER 6- COMBINED <i>RH</i> -ADAPTIVE FINITE ELEMENT METHOD.....	98
6.1. Introduction	99
6.2. ALE method	101
6.2.1. Mesh refinement	104
6.2.2. Remapping of state variables	107
6.3. Combined <i>rh</i> -adaptive method.....	108
CHAPTER 7- APPLICATIONS.....	113
7.1. Introduction	114
7.2. Numerical examples	114
7.2.1. Elastoplastic cylindrical cavity expansion	115
7.2.2. Biaxial test.....	117
7.2.3. Bearing capacity of soil under a strip footing	123
7.2.4. Undrained behaviour of a soil layer under a rigid footing.....	130
7.2.5. Cone penetration into a drained soil layer	139

7.2.6. Vertical cut.....	147
7.2.7. Indentation of a cylinder into a soil layer.....	151
7.2.8. Large deformation analysis of a footing on a two-layered undrained soil...	154
CHAPTER 8- CONCLUSION AND FUTURE WORK.....	1599
8.1. Summery	16060
8.2. Efficiency of h-adaptive method	1611
8.3. alternative error estimators	1622
8.4. Combined rh -adaptive method	1633
8.5. Future work	1633
REFERENCES.....	165

ABSTRACT

The finite element method (FEM) is extensively used in analysis of a wide range of nonlinear geotechnical problems. The finite element method can handle simple and complex constitutive soil models, and solve problems with complicated geometries and boundary conditions with reasonably accurate results. On the other hand, mesh distortion and entanglement of elements, occurring inevitably in failure zones with high stress/strain concentration, are main drawbacks of the common finite element solutions such as the Updated Lagrangian method. In addition, efficacious application of the method requires experience and a certain amount of trial and error, particularly when choosing an optimal time and spatial discretisation.

Adaptive finite element methods provide a means for obtaining more reliable solutions by continuously adjusting the discretisation in time and space according to the current solution. These procedures automatically refine, coarsen, or relocate a mesh to achieve a solution with a specified accuracy in an optimal fashion. Although a significant amount of research has been devoted to adaptive finite element analysis in solid mechanics, the application of adaptive methods has been less considered in nonlinear geotechnical problems due to the complexity. Modelling of problems in geomechanics is typically sophisticated due to nonlinear constitutive laws, large deformations, changing boundary conditions and time-dependent behaviour. A variety of adaptive finite element techniques have been developed to tackle nonlinear problems in solid mechanics. However, the

application of these methods to geomechanics is still a challenge. Amongst the various adaptive techniques, the r -adaptive and h -adaptive finite element methods are probably the most favoured and most established. r -adaptive finite element method attempts to eliminate the mesh distortion by refining the mesh in the finite element domain. On the other hand, h -adaptive finite element method is based on the idea of generating a new mesh by dividing the area of original elements where the interpolation should be improved to achieve higher accuracy or to avoid mesh distortion.

In this Thesis, the h -adaptive finite element technique will be employed to solve some complex geotechnical problems involving material nonlinearity, large deformation, changing boundary conditions and time-dependent nonlinearity. To achieve this, the main features of the technique including advanced mesh generation algorithms, error estimation methods and a procedure for remapping of state variables will be discussed and developed in company of a robust analysis program. The performance of the h -adaptive finite element method is then represented by considering the accuracy and efficiency of the method in solving some classical geomechanics problems such as the bearing capacity of footings, expansion of cavities, and the stability of slopes.

In addition, this Thesis will address the performance and the efficiency of alternative error estimation techniques for particular geotechnical applications involved with changing boundary conditions and inertia forces, such as static and dynamic penetration of an object into soil. Such problems are categorised as one of the most sophisticated problems of computational geomechanics due to their extreme nonlinearity.

This Thesis will also present a new and innovative combined adaptive method for tackling geotechnical problems with relatively large deformations. This robust method is based upon an elegant combination of the Arbitrary Lagrangian-Eulerian (ALE) method and the h -adaptive finite element method developed as a part of the Thesis. The proposed method takes advantage of r -refinement as well as h -refinement finite element techniques, and yet eliminates the individual drawback of each method.

PREFACE

The research work presented in the Thesis has been performed in the Discipline of Civil, Surveying and Environmental engineering, school of Engineering, at the University of Newcastle, Australia under supervision of Prof. Daichao Sheng from September 2007 to September 2011. During the term of candidature, a number of papers were published which are listed below:

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Nazem M, Kardani M, Carter JP, Sheng D, “A comparative study of error assessment techniques for dynamic contact problems of geomechanics”, *Computer and Geotechnics*, Vol 40, 62-73, 2012

Kardani M, Nazem M, Sheng D, Carter JP, “Large deformation analysis of geomechanics problem by combined rh -adaptive finite element method”, *Under co-authors’ review*

Kardani M, Nazem M, Abbo AJ, Sheng D, “A study of adaptive finite element methods in solving large deformation problems in geomechanics”, *IV European Conference on Computational Mechanics (ECCOMAS2010)*, Paris, France, 2010

Kardani M, Nazem M, Abbo AJ, Sheng D, “A comparative study of h -adaptive and r -adaptive finite element methods in geomechanics”, *Proceedings of 9th World*

Congress on Computational Mechanics and 4th Asian Pacific Congress on Computational Mechanics (WCCM2010), Sydney, Australia, 2010

Nazem M, Kardani M, Carter JP, Sheng D, “Application of h -adaptive FE method for dynamic analysis of geotechnical problems”, *Proceeding of the 2nd International Symposium on Computational Geomechanics (COMGEO2011)*, Cavtat-Dubrovnik, Croatia, 590-495, 2011

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Kardani M, Nazem M, Carter JP, “A combined rh -adaptive finite element method for geotechnical problems”, *Twelfth Pan American Congress of Applied Mechanics (PACAM 2012)*, Port of Spain, Trinidad, 2012
