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Scaling of mixed longitudinal-transverse velocity structure functions

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Abstract – Measurements in a turbulent round jet at a Taylor microscale Reynolds number R_λ of about 500 indicate that the mixed structure functions of u, the longitudinal velocity fluctuation, and v, the transverse velocity fluctuation, show a larger departure from the Kolmogorov scaling than the u structure functions, and a smaller departure than the v structure functions. Although the scaling exponents of the temperature structure functions are nearly equal to those of the v structure functions at a comparable order, the mixed structure functions of u and the temperature fluctuation θ exhibits a smaller departure from the Kolmogorov scaling than either u or θ structure functions at the same order. The difference between the scaling behaviour of these two mixed structure functions mainly reflects the difference in correlation between $(\partial u/\partial x)^2$ and either $(\partial v/\partial x)^2$ or $(\partial \theta/\partial x)^2$, implying that there is an important difference between temperature and velocity fields.

Introduction. – It is now well established that, over a scaling range (SR), moments of δu [$\equiv u(x+r)-u(x)$], δv [$\equiv v(x+r)-v(x)$] and $\delta\theta$ [$\equiv \theta(x+r)-\theta(x)$], where r is the separation between the two points in the longitudinal direction, exhibit an anomalous behaviour in that they deviate from the Kolmogorov [1] [K41] or K41-Obukhov [2] [K41-O49] scaling (*e.g.* [3–7]). We use SR rather than the inertial range (IR) since the latter applies strictly at infinite Reynolds numbers. Kolmogorov's [8] equation reduces to the 4/5 law in the IR. When the Reynolds number is finite, the departure from this law may be significant.

Assuming that the moments of $|\delta u|$, $|\delta v|$ and $|\delta\theta|$ in the SR are represented by power laws, viz.

$$\langle|\delta u|^m\rangle \propto r^{\zeta_u(m)}, \tag{1}$$

$$\langle|\delta v|^q\rangle \propto r^{\zeta_v(q)}, \tag{2}$$

$$\langle|\delta \theta|^n\rangle \propto r^{\zeta_\theta(n)}, \tag{3}$$

where angular brackets denote time averaging, the exponents $\zeta_u(m)$, $\zeta_v(q)$ and $\zeta_\theta(n)$ deviate from K41 [$\zeta_u(m) = m/3$, $\zeta_v(q) = q/3$] and K41-O49 [$\zeta_\theta(n) = n/3$] predictions. Independently of Obukhov [2], Corrsin [9] proposed the "$-5/3$" scaling for the temperature spectrum. The departure increases as m, q and n increase (*e.g.* [4,7,10]) and is larger for $\langle|\delta\theta|^n\rangle$ and $\langle|\delta v|^q\rangle$ than for $\langle|\delta u|^m\rangle$, *i.e.* $\zeta_\theta(n) < \zeta_u(m)$, $\zeta_v(q) < \zeta_u(m)$ for $m = n$ and $m = q$. Absolute values of structure functions are used hereafter to facilitate convergence of the moments when the order is odd. This approach has been used widely to study the scaling of the small scale motion, particularly via the Extended Self-Similarity (ESS) method [6]. The difference between conventional and absolute structure functions is not completely understood. Benzi *et al.* [6] assumed the scaling exponents to be the same for the two types of structure functions. However, there is no analytical underpinning of this assumption. Stolovitzky and Sreenivasan [11] found that there is a difference between the corresponding exponents. The absolute-valued structure functions have a larger scaling exponent than the conventional ones when the order n is large and odd [12]. Since the emphasis here is on comparing the scaling behaviour of $\langle|\delta u|^m|\delta v|^q\rangle$ with that of $\langle|\delta u|^m\rangle$, $\langle|\delta v|^q\rangle$, and $\langle|\delta u|^m|\delta\theta|^n\rangle$ for $q = n$, the use of the modulus does not seem too critical.

Dimensional arguments presented by Antonia and Van Atta [13] suggested that for a Prandtl number, Pr, near unity, the mixed velocity-temperature structure functions are given (in the IR) by

$$\langle(\delta u)^m(\delta\theta)^n\rangle = C_{mn} r^{(m+n)/3}\left\langle \varepsilon_r^{m/3-n/6}\chi_r^{n/2}\right\rangle, \tag{4}$$

where C_{mn} are universal constants which depend on the particular values of m and n. ε_r and χ_r are the energy and temperature dissipation rates, averaged over a linear dimension r.

Xu *et al.* [14] considered the IR scaling of the mixed longitudinal velocity-temperature structure functions $\langle|\delta u|^m|\delta\theta|^n\rangle$ defined by

$$\langle|\delta u|^m|\delta\theta|^n\rangle \propto r^{\zeta_{u\theta}(m,n)}. \tag{5}$$

They found that the mixed structure functions of u and θ show a smaller departure from K41-O49 scaling than either the velocity or temperature structure functions at the same order, *i.e.* $\zeta_{u\theta}(m,n) > \zeta_u(m) + \zeta_\theta(n)$.

Relatively little attention has been given to the scaling of $\langle(\delta u)^m(\delta v)^q\rangle$ or $\langle|\delta u|^m|\delta v|^q\rangle$ (absolute values are used here to facilitate the convergence for odd m or q). Assuming that, in the IR,

$$\langle|\delta u|^m|\delta v|^q\rangle \propto r^{\zeta_{uv}(m,q)}, \tag{6}$$

it is of interest to know how $\zeta_{uv}(m,q)$ compares with the sum $\zeta_u(m) + \zeta_v(q)$. If δu and δv are independent,

$$\zeta_{uv}(m,\ q) = \zeta_u(m) + \zeta_v(q). \tag{7}$$

Reviewing the spectra of u, v and θ, Sreenivasan [15] found that, in shear flows, the IR slopes of v and θ are similar and their values are likely to reach $-5/3$ only when R_λ exceeds about 1000. Sreenivasan [15] pointed out that the similarity between the slope of the temperature spectrum and that of the transverse velocity components is not a coincidence and suggested that, in non-homogeneous shear flows, "the scalar field attains a semblance of universality only if the velocity field in its entirety is universal". Using data in a rough wall boundary layer, Antonia and Smalley [16] noted that the slope of the temperature spectrum is quite close to that of the energy spectrum. This behaviour supports the close spectral analogy that is observed between the temperature and energy spectra (for a detailed discussion see chapt. 7 of Chassaing *et al.* [17]); these latter authors noted that the analogy breaks down when there is no mean shear or mean temperature gradient. In a turbulent wake, Antonia and Pearson [18] found that the scaling exponents inferred from the transverse velocity and temperature increments are nearly equal, *i.e.* $\zeta_\theta(n) \approx \zeta_v(q)$ for $n = q$. One may expect $\zeta_{uv}(m,q)$ and $\zeta_{u\theta}(m,n)$ to exhibit nearly the same behaviour for $q = n$. Here, the values of $\zeta_{uv}(m,q)$ have been obtained for $m + q \leqslant 9$ at a Taylor microscale Reynolds number R_λ of 495. The first aim of this paper is to test relation (7). The second is to examine the scaling of $\langle|\delta u|^m|\delta v|^q\rangle$ for $m + q = 3$, 6 and 9 where m and q are integers. For larger $(m + q)$, the convergence of $\langle|\delta u|^m\rangle$ and $\langle|\delta v|^q\rangle$ becomes questionable. The third is to compare the magnitude of $\zeta 1 \equiv \zeta_{uv}(2p,p)$, $\zeta 2 \equiv \zeta_{uv}(p,2p)$, $\zeta 3 \equiv \zeta_{uv}(3p,0)$, $\zeta 4 \equiv \zeta_{uv}(0,3p)(p \leqslant 4)$. In the IR, $\langle\delta u(\delta v)^2\rangle$ features in the asymptotic isotropic relation given by $\langle\delta u(\delta v)^2\rangle = -\frac{4}{15}\ \langle\varepsilon\rangle\, r$ [19,20]. Although there is no corresponding theoretical basis for $\langle|\delta u||\delta v|^2\rangle$, it is nonetheless of interest to study the scaling behaviour of $\langle(|\delta u||\delta v|^2)^p\rangle$. The exponents $\zeta 1 \equiv \zeta_{uv}(2p,p)$, $\zeta 2 \equiv \zeta_{uv}(p,2p)$, $\zeta 3 \equiv \zeta_{uv}(3p,0)$ and $\zeta 4 \equiv \zeta_{uv}(0,3p)$ $(p \leqslant 4)$ may help to clarify the scaling anomaly of the mixed structure functions of u and v. A final aim is to compare the scaling behaviour of $\langle|\delta u|^m|\delta v|^q\rangle$ with that of $\langle|\delta u|^m|\delta\theta|^n\rangle$ for $m + q = m + n$.

Experimental details and conditions. – The fluctuations u and v were measured on the axis of a turbulent round jet. The jet was supplied by a variable centrifugal blower through an axisymmetric nozzle with a 10 : 1 contraction ratio. The measurements were carried out at $x/d = 40$, where the flow is approximately self-preserving; this was inferred from the distributions of Reynolds normal and shear stresses. At this location, the longitudinal Taylor microscale Reynolds number R_λ is 495. The Kolmogorov length scale $\eta \equiv \nu^{3/4}\langle\varepsilon\rangle^{-1/4}$ ($\langle\varepsilon\rangle$ is the mean energy dissipation rate, estimated by assuming isotropy, *i.e.* $\langle\varepsilon\rangle = 15\nu\langle(\partial u/\partial x)^2\rangle$) and Kolmogorov velocity scale $U_k \equiv (\nu\langle\varepsilon\rangle)^{1/4}$ were 0.125 mm and 0.12 m/s at $x/d = 40$, respectively. An X-wire was used to measure the fluctuations u and v. The spanwise separation between the two wires of the X-probe was 1 mm. The sensing elements were made of 2.54 μm Pt-10% Rh Wollaston wire approximately 0.51 mm long. The hot wires were operated with constant temperature circuits at an overheat ratio of 1.5. The signals from the hot wires were digitised on a PC using 12 bit A/D converter at a sampling frequency of 12.6 kHz after examining the spectrum of the time derivative of u on a two-channel spectrum analyser (HP3582A); electronic noise first become noticeable at this frequency. The signals were subsequently transferred to a personal computer for further analysis.

Results and discussion. – Before the scaling exponents $\zeta_{uv}(m,q)$ are estimated, we need to identify the scaling range (SR). The extent of the SR affects the evaluation of the scaling exponents. Strictly, an inertial range (IR), as inferred from Kolmogorov's 4/5 law, is only likely to exist at very large Reynolds numbers. When the Reynolds number is finite, the extent of the SR depends on the magnitude of the Reynolds number (*e.g.* [21,22]). In the present study, an indication of the location and extent ($26 \leqslant r^* \leqslant 253$) of the SR is provided by the distributions of $r^{*\,-2/3}\langle(\delta u^*)^2\rangle$ and $r^{*\,-1}\langle(\delta u^*)^3\rangle$, shown in fig. 1 (an asterisk denotes normalisation by η and U_k). Included in the figure are $r^{*\,-1}\langle\delta u^*(\delta v^*)^2\rangle$, $r^{*\,-1}\langle|\delta u^*||\delta v^*|^2\rangle$ and $r^{*\,-1}\langle|\delta u^*|^3\rangle$. Ideally, the third-order mixed structure function of u and v should scale with r, *i.e.* $\langle\delta u^*(\delta v^*)^2\rangle = -4r^*/15$, when the IR exists. Figure 1 shows that the extent of the SR differs between $\langle(\delta u)^2\rangle$, $\langle(\delta u)^3\rangle$, $\langle\delta u(\delta v)^2\rangle$, $\langle|\delta u|^3\rangle$ and $\langle|\delta u||\delta v|^2\rangle$. The use of the modulus generally widens the SR (*e.g.* [6]). Figure 1 indicates that for $\langle|\delta u||\delta v|^2\rangle$, the extent of the

Table 1: Scaling exponents $\zeta_{uv}(m,q)$. The values in the brackets are for $\zeta_u(m)+\zeta_v(q)$.

m / q	0	1	2	3	4	5	6	7	8	9
0		0.36	0.72	1.02	1.30	1.56	1.80	2.03	2.27	2.50
1	0.31		0.98(1.03)			1.76(1.87)			2.47(2.58)	
2	0.62	0.93(0.98)			1.70(1.92)			2.41(2.65)		
3	0.86			1.64(1.88)			2.33(2.66)			
4	1.07		1.57(1.79)			2.24(2.63)				
5	1.25	1.49(1.61)			2.14(2.55)					
6	1.40			2.04(2.42)						
7	1.54		1.95(2.26)							
8	1.66	1.88(2.02)								
9	1.78									

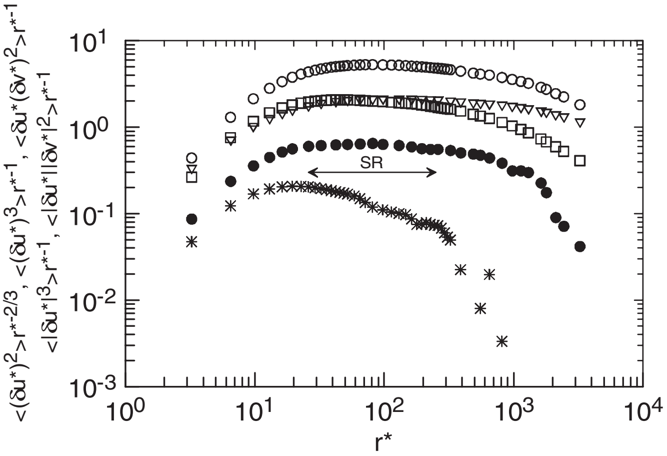

Fig. 1: Kolmogorov-normalized second- and third-order structure functions at $R_\lambda = 495$. $\triangledown$, $r^{*\,-2/3}\left\langle(\delta u^*)^2\right\rangle$; $\bullet$, $r^{*\,-1}\left\langle(\delta u^*)^3\right\rangle$; $*$, $r^{*\,-1}\left\langle\delta u^*(\delta v^*)^2\right\rangle$; $\circ$, $r^{*\,-1}\left\langle|\delta u^*|^3\right\rangle$; $\square$, $r^{*\,-1}\left\langle|\delta u^*||\delta v^*|^2\right\rangle$.

SR is much larger than for $\left\langle\delta u(\delta v)^2\right\rangle$, in agreement with Benzi *et al.* [6]. In the SR, $r^{*\,-2/3}\left\langle(\delta u^*)^2\right\rangle$ is about 1.95, while $r^{*\,-1}\left\langle(\delta u^*)^3\right\rangle$ and $r^{*\,-1}\left\langle\delta u^*(\delta v^*)^2\right\rangle$ are about 0.67 and 0.2, respectively. The latter two values are smaller than Kolomogorov's asymptotic values of 4/5 and 4/15. It should be recalled that the non-stationarity of $\left\langle(\delta u)^2\right\rangle$ is neglected in Kolmogorov's [8] equation. This assumption is only tenable when R_λ is very large or r^* is very small. The non-stationarity, which is included in the Karman-Howarth equation or in the corresponding structure function equation [21] needs to be considered when R_λ is finite, especially outside the dissipative range. Antonia and Burattini [23] showed, also for homogeneous isotropic turbulence, that the maximum value of $r^{*\,-1}\left\langle(\delta u^*)^3\right\rangle$ should approach 4/5 only slowly, unless the turbulence is forced.

Estimates of $\zeta_{uv}(m,q)$ (eq. (6)), including $\zeta1$, $\zeta2$, $\zeta3$ and $\zeta4$ [$\zeta1\equiv\zeta_{uv}(2p,p)$, $\zeta2\equiv\zeta_{uv}(p,2p)$, $\zeta3\equiv\zeta_{uv}(3p,0)$, $\zeta4\equiv\zeta_{uv}(0,3p)(p\leqslant 4)$], were determined from least-squares linear regressions to $\log\langle|\delta u|^m|\delta v|^q\rangle$, $\log\left\langle(|\delta u|^2|\delta v|)^p\right\rangle$, $\log\left\langle(|\delta u||\delta v|^2)^p\right\rangle$, $\log\left\langle(|\delta u|^3)^p\right\rangle$ and $\log\left\langle(|\delta v|^3)^p\right\rangle$ *vs.* $\log(r)$ in the SR. This is usually referred to as the direct method [7]; for each regression, the squared correlation coefficient was greater than 0.997. The extended self-similarity (ESS) method [6], *i.e.* plotting $\log\langle|\delta u|^m|\delta v|^q\rangle$ against $\log\left\langle|\delta u||\delta v|^2\right\rangle$, was also used. The exponents $\zeta_{uv}(m,q)$ (eq. (6)) estimated by the ESS method are about 3% smaller than those estimated by the direct method. The values (table 1) of $\zeta_{uv}(m,q)$ for $m+q=3$, 6 and 9 were obtained using the direct method. The uncertainty for the scaling exponents in table 1 was estimated to be in the range ±0.01 for $m+q=1$ to ±0.06 for $m+q=9$. The values of $\zeta_{uv}(0,q)$ [$\equiv\zeta_v(q)$] are smaller than those of $\zeta_{uv}(m,0)$ [$\equiv\zeta_u(m)$] for $m=q$, suggesting that $\langle|\delta u|^m\rangle$ is more anomalous than $\langle|\delta v|^q\rangle$. This is in agreement with previous measurements (*e.g.* [4,18]). The values of $\zeta_{uv}(6,0)$ and $\zeta_{uv}(2,4)$ are 1.80 and 1.57, respectively, implying values for the intermittency exponents $\mu\equiv 2-\zeta_{uv}(6,0)$ and $\mu_v\equiv 2-\zeta_{uv}(2,4)$ of 0.20, 0.43 respectively, indicating that v is more intermittent than u. K41 predicts that $\zeta_u(m)=m/3$ and $\zeta_v(q)=q/3$. It follows that $\zeta_{uv}(m,q)=\zeta_u(m)+\zeta_v(q)=(m+q)/3$, if δu and δv are statistically independent. The present results depart from K41, *i.e.* $\zeta_{uv}(m,q)\neq\zeta_u(m)+\zeta_v(q)=(m+q)/3$. For the non-zero values of m and q,

$$\zeta_{uv}(m,\ q)<\zeta_u(m)+\zeta_v(q), \tag{8}$$

i.e. $\langle|\delta u|^m|\delta v|^q\rangle$ is more anomalous than $\langle|\delta u|^m\rangle$ and $\langle|\delta v|^q\rangle$. By comparing (8) with (7), the inference is that the correlation between δu and δv cannot be ignored.

To explain the difference between $\zeta_u(m)$ and $\zeta_v(q)$ at finite R_λ, Chen *et al.* [24] suggested that δu scales with ε, whereas δv scales with the enstrophy, ω^2. At very large R_λ, one would expect ε_r^u [$\equiv 15\nu(\partial u/\partial x)_r^2$] and ε_r^v [$\equiv 7.5\nu(\partial v/\partial x)_r^2$] to be perfectly correlated. For small to moderate R_λ, we hypothesise that δu scales with ε_r^u and δv scales with ε_r^v. Assuming that both the probability density functions (pdfs) of ε_r^u, ε_r^v, and the joint pdf of ε_r^u,

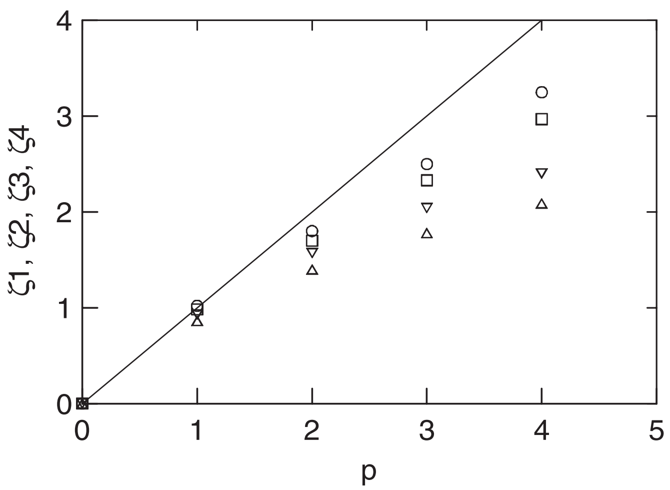

Fig. 2: Scaling exponents $\zeta 1 \equiv \zeta_{uv}(2p,p)$, $\zeta 2 \equiv \zeta_{uv}(p,2p)$, $\zeta 3 \equiv \zeta_{uv}(3p,0)$, $\zeta 4 \equiv \zeta_{uv}(0,3p)(p \leqslant 4)$. □, ζ1; ▽, ζ2; ○, ζ3; △, ζ4. —, K41 ($\zeta j = p$, $j = 1$, 2, 3 and 4).

ε_r^v are lognormal [10,25,26],

$$\Delta\zeta_{uv}(m,q) \equiv \zeta_{uv}(m,q) - \zeta_u(m) - \zeta_v(q) = \frac{\mu^{1/2}}{18}(\mu^{1/2} - 3\rho\mu_v^{1/2})mq, \tag{9}$$

where ρ is the correlation coefficient between the centred variables $\ln\varepsilon_r^u$ and $\ln\varepsilon_r^v$. The average value of ρ is about 0.78 in the SR, implying a reasonably strong correlation between ε_r^u and ε_r^v. Relation (9) shows that the magnitudes of μ, μ_v and ρ should affect the difference between the scaling exponents of $\zeta_{uv}(m,q)$ and those of $\zeta_u(m) + \zeta_v(q)$, and the sign of $\Delta\zeta_{uv}(m,q)$ depends on the relative magnitudes of $\mu^{1/2}$ and $3\rho\mu_v^{1/2}$. In the present case, $\mu = 0.20$, $\mu_v = 0.43$ and $\rho = 0.78$ and eq. (9) yields $\Delta\zeta_{uv}(m,q) < 0$, consistent with our observation (Relation (8)).

It is expected that for the same value of $m+q$, the larger the order m, the smaller the departure of $\zeta_{uv}(m,q)$ from K41 since $\zeta_u(m) > \zeta_v(m)$. The scaling exponents ζ1, ζ2, ζ3 and ζ4 ($\zeta 1 \equiv \zeta_{uv}(2p,p)$, $\zeta 2 \equiv \zeta_{uv}(p,2p)$, $\zeta 3 \equiv \zeta_{uv}(3p,0)$, $\zeta 4 \equiv \zeta_{uv}(0,3p)(p \leqslant 4)$) are plotted in fig. 2. The figure indicates that $\zeta 3 > \zeta 1 > \zeta 2 > \zeta 4$, ζ2 being closer to ζ4 than ζ3. The solid line in the figure represents K41 ($\zeta j = p$, $j = 1$, 2, 3 and 4). All the values of ζ1, ζ2, ζ3 and ζ4 depart from K41. The departure of $\langle(|\delta u|^3)^p\rangle$ is much smaller than that of $\langle(|\delta u|^2|\delta v|)^p\rangle$, $\langle(|\delta u||\delta v|^2)^p\rangle$ and $\langle(|\delta v|^3)^q\rangle$ for larger values of $(m+q)$ $(\equiv 3p)$. $\langle(|\delta v|^3)^q\rangle$ exhibits the largest departure.

The observation here is different from that of Xu *et al.* [14]. They found that

$$\zeta_{u\theta}(m,\ n) > \zeta_u(m) + \zeta_\theta(n). \tag{10}$$

The present values of $\zeta_u(m)$ are in good agreement with those of Xu *et al.* [14]. The present value of $\zeta_v(q)$ are nearly equal to those of $\zeta_\theta(n)$ [14] for $q = n$. The observed difference between the scaling behaviour of $\langle|\delta u|^m|\delta v|^n\rangle$ and $\langle|\delta u|^m|\delta\theta|^n\rangle$ is ascribed to the difference between the correlation coefficients $\rho_{\ln\varepsilon_r^u,\ln\varepsilon_r^v}$ and $\rho_{\ln\varepsilon_r^u,\ln\chi_r}$ $(\chi_r \equiv 3\alpha(\partial\theta/\partial x)_r^2)$. The correlation coefficient $\rho_{\ln\varepsilon_r^u,\ln\chi_r}$ between the centred variables $\ln\varepsilon_r^u$ and $\ln\chi_r$ is about 0.15 [14], which is much smaller than the present value (0.78) for the correlation coefficient $\rho_{\ln\varepsilon_r^u,\ln\varepsilon_r^v}$ between the centred variables $\ln\varepsilon_r^u$ and $\ln\varepsilon_r^v$. In terms of eq. (9), results (8) and (10) reflect the previous large difference in ρ. For grid turbulence, there seems to be a $-5/3$ scaling in the temperature spectrum but not in the velocity spectrum (*e.g.* [27]). For homogeneous isotropic turbulence, Burattini and Antonia [28] found that $-\langle(\delta u^*)(\delta\theta^*)^2\rangle$ approaches 4/3 much more rapidly than $-\langle(\delta u^*)^3\rangle$ approaches 4/5. Figure 1 and that of Xu *et al.* [14] indicate that $\langle(\delta u^*)(\delta\theta^*)^2\rangle$ has a better scaling than $\langle(\delta u^*)(\delta v^*)^2\rangle$. This seems to support the previous trend. Venkataramani and Chevray [29] found, for grid turbulence with a mean temperature gradient, that the temperature spectrum displays a more extensive scaling than the velocity spectrum under the same conditions. For shear flows, the temperature spectrum has, as for grid turbulence, a more extensive scaling range than the longitudinal velocity spectrum under the same conditions (*e.g.*, [15]). This is consistent with the present finding and Warhaft's [30] observation that temperature behaves differently to velocity.

Conclusions. – The scaling exponents of the transverse velocity structure functions are smaller than those of the longitudinal velocity structure functions. The trend is in agreement with previous measurements. The mixed *u-v* structure functions show a larger departure from K41 scaling than the *u* structure functions, and a smaller departure from K41 scaling than the *v* structure functions. Although the scaling exponents of the temperature structure functions are nearly equal to those of the *v* structure functions at the same order, the mixed structure functions of *u* and *v* exhibit a different scaling behaviour from that of the mixed structure functions of *u* and θ. This behaviour reflects the difference in correlation between $(\partial u/\partial x)^2$ and either $(\partial v/\partial x)^2$ or $(\partial\theta/\partial x)^2$.

The data analysed here were obtained on the jet axis; away from the axis, the anisotropy should increase due to the increasing shear (*e.g.* [31]) and affect the scaling range. The combined effect of the shear and the Reynolds number on the behaviour of the scaling range merits a future study.

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